Nash Equilibrium

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1 Coordination and Anti-Coordination

1.1 Coordination

Consider a scenario where you and a friend go to Disneyland together but lost each other. There are two possible meeting points, (Matterhorn or Space Mountain) and the players have to decide independently where to meet. They do not care where they meet, but they must meet together. A payoff table for this scenario is summarized below:

		2	
		M	S
1	M	5,5	0,0
	S	0,0	5,5

We note that for player 1, we have the following responses:

$$\begin{cases} M & \text{if } s_2 = M \\ S & \text{if } s_2 = S \end{cases}$$

It is clear that this game does not have any weakly or strongly dominated strategies, and every strategy is within the set of best responses. There is no elimination of any dominated strategy, and players therefore have ultimately no way of directly coordinating.

2 Nash Equilibrium

Definition 2.1. *Strategy profile $s^* = \{s_i^*..s_n^*\}$ is a Nash Equilibrium if $\forall i : s_i^* \in BR_i(s_j^*)$, also denoted:*

$$u_i(s_i^*, s_j^*) \geq u_i(s_i', s_j^*) \ \forall \ s_i' \in S_i$$

Using this definition, we see that (M,M) and (S,S) are both in a Nash equilibrium in the above example.

2.1 Nash Equilibrium and Dominant Strategies

We have a following table regarding rationality:

Players	**Will play BR to**
are Rational	Some beliefs
have common knowledge of Rat.	BR to BR's (rationalizable)
Rat. and somewhat coord.	Correct beliefs (Nash Eq.)

2.2 Justification

Question 2.1. How do we justify the concept of Nash Equilibrium and correct beliefs?

There are multiple reasons to justify the fact that your beliefs of the other players is correct:

- **Learning**: Suppose that after repetitions, players eventually settle on an individual strategy s^*. Once this is expected to be constant, there is no incentive to deviation from the already settled strategy.
- **Self-Enforcing Agreement**: If players initially agree to play a NE, there exist no incentive to ever deviate from the NE. This occurs *iff.* in a NE situation.

3 Examples

3.1 Prisoner's Dilemma

		2 C	2 D
1	C	2,2	0,3
1	D	3,0	1,1

It is clear that (D, D) is the unique Nash Equilibrium in this scenario. If a call is made to both "agree", then each player has the incentive to just defect, so (C, C) can't be a NE.

3.2 Chicken

		2 S	2 C
1	S	-5,-5	5,0
1	C	0,5	2,2

In this game, (C, S) and (S, C) are both NE's.

3.3 Rock, Paper, Scissors

		2 R	2 P	2 C
1	R	0,0	-1,1	1,-1
1	P	1,-1	0,0	-1,1
1	S	-1,1	1,-1	0,0

There is **no** pure strategy Nash Equilibrium. However, there is a mixed strategy Nash Equilibrium.

4 Theorems about Nash Equilibriums

Theorem 4.1 (Rationalizability)**.** *Every strategy s_i^* in a Nash Equilibrium, $s^* = \{s_1^*..s_n^*\}$ is rationalizable*

Proof Assume that the strategy profile $s^* = \{s_i^*..s_n^*\}$ has survived k rounds of I.D., s.t. $s_i^* \in BR_i^k$. By induction, we must show that $s_i^* \in BR_i^{k+1}$ by finding σ_j with positive probability only to strategy $s_j \in BR_j^k \wedge s_i^* \in BR_i(\sigma_j)$. A belief $\sigma_j := P(s_j^*) = 1 \wedge P(\neq s_j^*) = 0$ has these properties $\because s_j^* \in BR_j^k \wedge s^* = \text{NE}$. ■

Theorem 4.2 (Dominance Solvable)**.** *Suppose a game is dominance solvable and that s_i^R is the unique rationalizable strategy $\forall i$. Then, $\{s_1^R..s_n^R\}$ is the unique Nash Equilibrium.*

Proof To Add Later ■

This theorem shows that a Nash Equilibrium is a stronger concept than rationalizability, formalized by saying, $\{s_1^*..s_n^*\} \subseteq BR_i^k$.