Extensive Form Games

Felix Jen

May 3, 2018

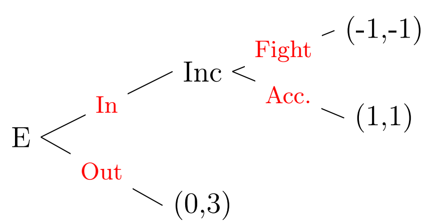
1 Extensive Form Games

In many situations, games unfold simultaneously, warranting the use of *dynamic games*. However, some things unfold over a period of time, in a sequence, making an *extensive game*.

Definition 1.1 (Extensive Form Game)**.** *An extensive form game consists of*

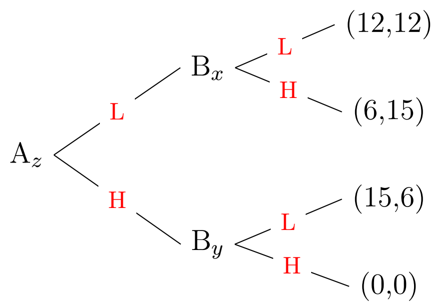
- *A set of players* $I \in \{1, 2, 3..n\}$
- *A game tree with nodes* x, y, z *and branches*
  - *Each action node (a node before the end)* x *is labeled with a player* i
  - *From each action node* x*,the possible actions* $a_i(x)$ *of player* $i(x)$ *are represented by the branches leaving the node*
  - *Some of* i*'s action nodes can be circled, meaning that they belong to the same information set*
- *Payoffs for each player for each terminal node* $t : u_i(t), i \in I$

Example 1.1 (Simple Entry Game)**.** Imagine a situation if there are two firms in the industry and one firm wants to enter the firm. The incumbent has the power to start a pricing war, etc...



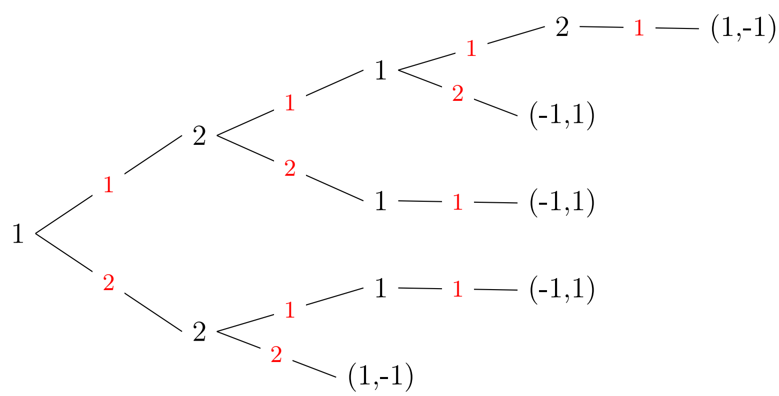
□

Example 1.2 (Simple Stackelberg Duopoly)**.** Suppose there is some firm in the industry that is a leader and can choose the quantity it wants to produce. The second firm can response to that quantity and choose its own.



□

Example 1.3 (Game of Nim)**.** Suppose there are 2 players that start with a $n = 4$ matches. Player 1 starts and each chooses whether to pick up one or 2 matches and the players alternate. The player picking up the last match loses.



□

Definition 1.2 (Strategy)**.** *A strategy for i in an extensive form game specifies i's choices at each of i's action nodes*

Remark 1.1. Each extensive form game can be analyzed as a static strategic form game. In particular, we can define a Nash Equilibria in an extensive form game △

Example 1.4 (Market Entry Game)**.** Consider the market entry game. A payoff matrix can be created:

		E	
		In	Out
Inc	F	-1,-1	3,0
	A	1,1	3,0

A pure strategy Nash Equilibria can be found. Note that (A,In) and (F,Out) are both Nash Equilibria. □

Example 1.5 (Stackelberg Nash)**.** We can define a payoff matrix as:

		B_2			
		H(x),H(y)	H(x),L(y)	L(x),H(y)	L(x),L(y)
A_1	L	6,15	6,15	12,12	12,12
	H	0,0	15,6	0,0	15,6

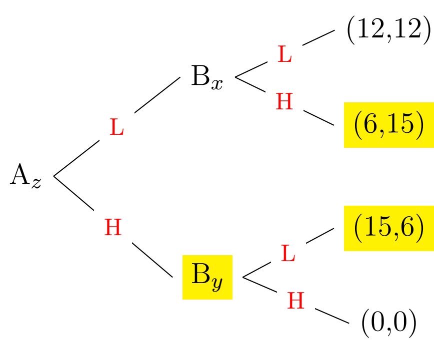
We can find our Nash Equilibria as H(x),H(y), H(x),L(y), and L(x),L(y). It's important to note that two of those Nash Equilibria make little sense in this game. Firm B choosing H is not very credible. At Node y, if firm B was making a choice, it should choose L. Therefore, the threat of H is incredible. As well, at node x, firm B should choose H, instead of L. Therefore, there needs to be a more complete definition of an equilibrium in an extensive form game.

□

Definition 1.3 (Backwards Induction)**.** *A backward induction solution is a strategy profile that satisfies:*

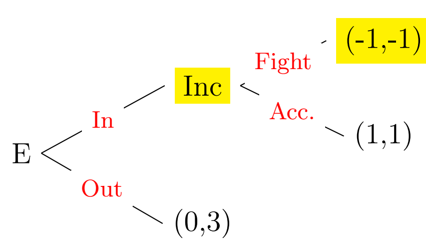
- *At last action nodes, each player picks an optimal choice*
- *At penultimate nodes, each player picks an optimal choice, taking later choices as given*
- *... until initial node*

Example 1.6 (Backwards Induction with Stackelberg)**.** We can perform a backwards induction as follows:



□

Example 1.7 (Simple Entry Game)**.** We can perform a backwards induction as follows:



□

Theorem 1.1 (Zermelo's Theorem)**.** *Every finite game with complete information has a backwards induction solution. Moreover, if all payoffs differ, the backwards induction solution is unique.*

Example 1.8 (Game of Nim)**.** Varying the number of matches in a game of Nim, we have a table of possible results:

N	P_1	P_2
2	+	-
3	+	-
4	-	+
5	+	-
6	+	-
...	...	...

□

In this table, whichever player has a (+) has an enforcible winning strategy.

Theorem 1.2. *In zero-sum games with payoffs (1,-1) or (-1,1), one player must have a winning strategy.*