Cournot and Bertrand Competition

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1 Cournot Competition

Consider a situation involving the decision of OPEC countries with how much oil to produce. Note that the two *countries* are oligopolistic and are able to influence market prices. We have the following conditions:

- Players are s.t. $i = 1..n = 2$
- Actions are defined as $q_i \in A_i = [0, \infty)$
- Utility is equivalent to profit s.t. $u_(q_1..q_n) = p(Q) - c_1(q_i)$ where $Q := \sum_j q_j$ which is the total amount of q produced
- $p(Q) = 1000 - Q$
- $c_i(q_i) = 100q_i$
- Given $p(Q)$ and $c_i(q_i)$, $u_i(q_i, q_j) = (900 - q_i - q_j)(q_i)$

1.1 Nash Equilibrium Solution

To find a NE, we find $(q_1^*, q_2^*) : q_1^* \in BR(q_2^*) \ \wedge \ q_2^* \in BR(q_1^*)$. Therefore, we have the following objective:

$$\max_{q_i} u_i(q_i, q_j) = \max_{q_i}(900 - q_i - q_j)q_i$$

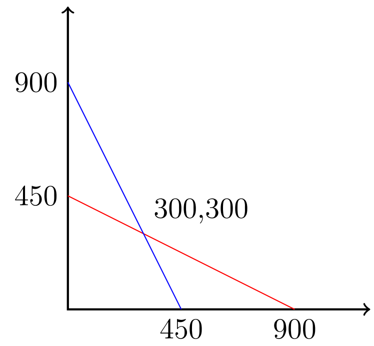
Given that we know $MR = MC \ \forall \ q$, we can take the F.O.C. to result in:

$$\frac{\partial u_i}{\partial q_i} = (900 - q_1 - q_j) - q_i = 0 \implies BR_i(q_j) = \frac{900 - q_j}{2}$$

Therefore, given the symmetry of the game, we have both:

$$BR_i(q_j) = \frac{900 - q_j}{2} \ \wedge \ BR_j(q_i) = \frac{900 - q_i}{2}$$

Note that following symmetry:



We can solve the system to be $q^* = 300$ given the inherent symmetry in the game. Note that this is the unique NE for this game. The payoffs of this game are ultimately calculated as:

$$U = 300(900 - 600) = 90000$$

1.2 Cartel Situation

Imagine a situation where the firms opt to form a cartel. Both firms choose q^* and split profits equally. In this situation, the firms would optimize to:

$$\max_Q Q(1000 - Q) - 100Q \implies 900 - 2Q = 0 \to Q^* = 450$$

In this situation, payoffs would be:

$$Q^*(900 - Q^*) \implies 202,500$$

1.3 Results Table

Taking things further, we can generate the following table of outcomes:

	# of Firms	Total Quan	Price	Total Profit
Cartel/Monopoly	1	450	550	202,500
Duopoly	2	600	300	180,000
Oligopoly				
Competitive	∞	900	100	0

2 Bertrand Model

Imagine two firms which opt to choose optimal price. Note the following conditions:

- Players $i \in \{1..2\}$
- Strategy set $S_1 = S_2 = [0, 1000] : p_1 \wedge p_2 \in S$
- Payoffs are s.t.
  - Constant MC s.t. $c_1 = c_2 = 100$
  - Consumers choose solely based on price

- Demand is s.t. $Q(p) = 1000 - \underline{p} \mid \underline{p} := \min(p_1, p_2)$

Therefore, we can define payoff to be:

$$\pi_1(p_1, p_2) = \begin{cases} (p_1 - c_1) \cdot (1000 - p_1) & p_1 < p_2 \\ (p_1 - c_1) \cdot \frac{1000 - p_1}{2} & p_1 = p_2 \\ 0 & p_1 > p_2 \end{cases}$$

This is symmetric with respect to p_2.

2.1 Nash Equilibrium Solution

Note that:

$$BR_i(p_2) = \begin{cases} 550 & p_2 > 550 (\text{ Monopoly Price}) \\ p_2 - \lim_{\epsilon \to 0} \epsilon & 100 < p_2 \leq 550 \\ p_1 \geq p_2 & p_2 = 100 \\ p_1 > p_2 & p_2 < 100 \end{cases}$$

The same best response is the same for the other player. We can see that if any player chooses $p < 100$, the other firm can do better just by playing 100. If both players chooses $p > 100$, the other firm can do better by slightly undercutting the other. If $p_i > p_j \geq 100$, then p_j can get more profits by pricing closer to p_1. **Therefore, the only unique Nash Equilibrium is** $p_1 = p_2 = 100$.

2.2 Revelations

Note that in Cournot and Bertrand model, we did not change any of the game's structure. However, the Cournot, the firms would still make a profit, whereas for Bertrand, two firms is enough to achieve the zero-profit condition. This is really not true in real life for multiple reasons:

- Differentiated products
- Firms are capacity constrained
- Firms have different costs
- Firms use the "match lowest price" condition

2.3 Differentiated Products

Let's reduce the assumption that firms produce fully homogenized products, but instead, consumers are labeled by $x \in \{0..1\}$ and that consumer x receives a utility s.t.:

$$u_x = \begin{cases} 10 - p_1 - x & \textbf{Buys from Firm 1} \\ 10 - p_2 - (1 - x) & \textbf{Buys from Firm 2} \end{cases}$$

2.4 Match Lowest Price

Suppose that there exists a situation where if a customer presents the competitor's lower price, then the firm is obligated to sell at the lower price. In this situation, we have the following payoffs:

$$\pi(p_1, p_2) = (p^{\min} - 100) \cdot \frac{1000 - p^{\min}}{2}$$

Therefore, the claim is that any pair of identical $p_1 = p_2 \in [100, 500]$ is a NE.